Investigation • Where Will They Meet?

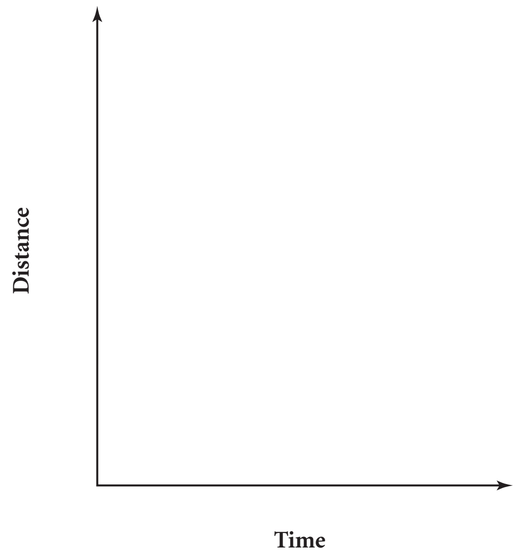
Name ______________________________ **Period** __________ **Date** ______________

You will need: one motion sensor, a tape measure and chalk to make a 6-meter line segment

In this investigation you'll solve a system of simultaneous equations to find the time and distance at which two walkers meet.

Suppose that two people begin walking in the same direction at different average speeds. The faster walker starts behind the slower walker. When and where will the faster walker overtake the slower walker?

Step 1 Sketch a graph showing both walks. Which line represents the faster walker?



Now act out the walk.

Step 2 Mark a 6 m segment at 1 m intervals. In your group, designate Walkers A and B, a timekeeper, and a recorder.

Step 3 Practice these walks: Walker A starts at the 0.5-m mark and walks toward the 6-m mark at a speed of 1 m/s. Walker B starts at the 2-m mark and walks toward the 6-m mark at 0.5 m/s.

> **Procedure Note**
>
> The timekeeper counts each second out loud. The walkers walk at the given speeds by noting their positions on the marked segment. The recorder uses a motion sensor to measure the time and position of each walker.

Step 4 When the walkers can follow the walk instructions accurately, record and download the motion of each walker as a separate event. First record Walker A's motion with the motion sensor. Download Walker A's data to a graphing calculator and move it to other lists. [▶ See **Calculator Note 3B: Collecting Distance Data Using the EasyData App** and **1B: Entering Lists**. ◀] Then record Walker B's motion, and download these data to the same graphing calculator.

Next you'll model the walks with a system of equations.

Step 5 Find an equation to model the data for each of the two walkers.

Step 6 On your calculator, graph the two equations on the same axes with both sets of data. Find the approximate point where the lines intersect.

Step 7 Explain the real-world meaning of the intersection point in Step 6.

Step 8 Check that the coordinates of the point of intersection satisfy both of your equations.

Next you'll consider what happens under different conditions.

Step 9 Suppose that Walker A walks faster than 1 m/s. How is the graph different? What happens to the point of intersection?

Step 10 Suppose that two people walk at the same speed and direction from different starting marks. What does this graph look like? What happens to the solution point?

Step 11 Suppose that two people walk at the same speed in the same direction from the same starting mark. What does this graph look like? How many points satisfy this system of equations?

Name ______________________ **Period** __________ **Date** ____________

In this investigation you'll solve a system of simultaneous equations to find the time and distance at which two walkers meet.

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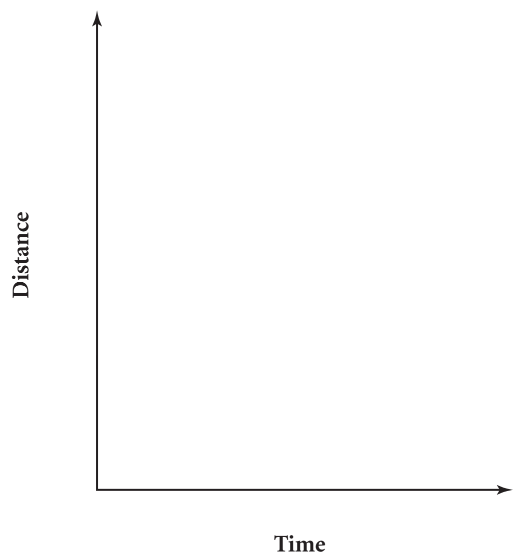


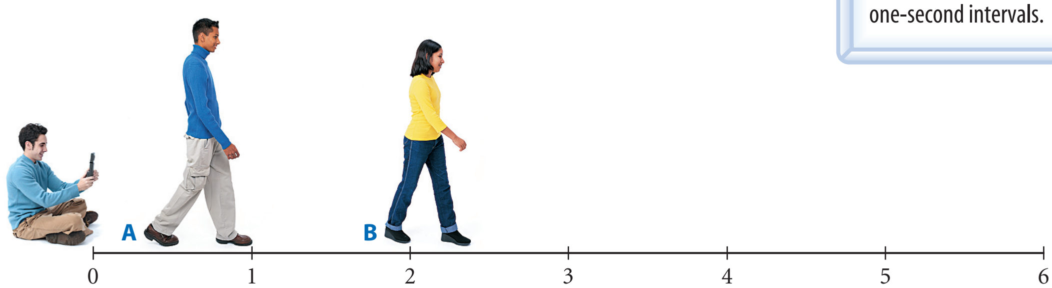
This is the procedure that would be used to gather data for the walk.

Step 2 A 6 m segment is marked at 1 m intervals. Group members are assigned these roles: Walkers A and B, a timekeeper, and a recorder.

Step 3 Walker A starts at the 0.5-m mark and walks toward the 6-m mark at a speed of 1 m/s. Walker B starts at the 2-m mark and walks toward the 6-m mark at 0.5 m/s.

Procedure Note

The timekeeper counts each second out loud. The walkers walk at the given speeds by noting their positions on the marked segment. The recorder measures the time and position of each walker at one-second intervals.



Step 4 Use the information in Step 3 to determine the distance each walker is from the starting point (0) of the 6-m segment at one-second intervals. Record the data in the table.

Time (s)	Walker A distance (m)	Walker B distance (m)
0		
1		
2		
3		
4		
5		
6		

Next you'll model the walks with a system of equations.

Step 5 Find an equation to model the data for each of the two walkers.

Step 6 On your calculator, graph the two equations on the same axes with both sets of data. Find the approximate point where the lines intersect.

Step 7 Explain the real-world meaning of the intersection point in Step 6.

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Investigation • All Tied Up

Name ________________________ **Period** __________ **Date** ______________

You will need: two ropes of different thickness, both about 1 meter long, a meterstick or tape measure, a 9-meter-long thin rope (optional), a 10-meter-long thick rope (optional)

In this investigation you'll work with rope lengths and predict how many knots it would take in each rope to make a thicker rope the same length as a thinner one.

First you'll collect data and write equations.

Step 1 Measure the length of the thinner rope without any knots. Then tie a knot and measure the length of the rope again. Continue tying knots until no more can be tied. Knots should be of the same kind, size, and tightness. Record the data for number of knots and length of rope in the thin rope column of the table.

Number of knots	Thin rope length (cm)	Thick rope length (cm)
0		
1		
2		
3		
4		
5		
6		

Step 2 Define variables and write an equation in intercept form to model the data you collected in Step 1. What are the slope and y-intercept, and how do they relate to the rope?

Step 3 Repeat Steps 1 and 2 for the thicker rope. Record the data in the thick rope column of the table.

Step 4 Suppose you have a 9-meter-long thin rope and a 10-meter-long thick rope. Write a system of equations that gives the length of each rope depending on the number of knots tied.

Next you'll analyze the system to find a meaningful solution.

Step 5 Solve this system of equations using the substitution method.

Step 6 Select an appropriate window setting and graph this system of equations. Estimate coordinates for the point of intersection to check your solution. Compare this solution with the one from Step 5.

Step 7 Explain the real-world meaning of the solution to the system of equations.

Step 8 What happens to the graph of the system if the two ropes have the same thickness? The same length?

Name ______________________________ **Period** __________ **Date** ______________

In this investigation you'll work with rope lengths and predict how many knots it would take in each rope to make a thicker rope the same length as a thinner one.

Step 1 The experiment began with a thin rope 1 meter long. After a knot was tied in the rope, the length was measured. The knot-tying and measuring continued until no more knots could be tied. Knots were of the same kind, size, and tightness. The data were recorded in the thin rope column of the table.

Number of knots	Thin rope length (cm)	Thick rope length (cm)
0	100	100
1	94	89.7
2	88	78.7
3	81.3	68.6
4	75.7	57.4
5	69.9	47.8
6	63.5	38.1

Step 2 Define the variables and write an equation in intercept form to model the data from tying the thin rope. What are the slope and the y-intercept, and how do they relate to the rope?

Step 3 The process used in Step 1 was repeated, this time using the thick rope. Those data were recorded in the table. Define the variables and write an equation in intercept form to model the data from tying the thick rope. What are the slope and the y-intercept, and how do they relate to the rope?

Step 4 Suppose you have a 9-meter-long thin rope and a 10-meter-long thick rope. Write a system of equations that gives the length of each rope depending on the number of knots tied.

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Step 5 Solve this system of equations using the substitution method.

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Step 8 What happens to the graph of the system if the two ropes have the same thickness? The same length?

Investigation • Paper Clips and Pennies

Name ____________________ **Period** __________ **Date** ______________

You will need: three paper clips, several pennies, an 8.5-by-11-inch sheet of paper

In this investigation you'll create a system of equations by using paper clips and pennies as variables.

Step 1 Lay one paper clip along the long side of the paper. Then add enough pennies to complete the 11-inch length.

Step 2 Use C for the length of one paper clip and P for the diameter of one penny. Write an equation in standard form showing your results.

Step 3 Now you'll write the other equation for the system. Lay two paper clips along the shorter edge of your paper, and then add pennies to complete the 8.5-inch length.

Step 4 Using the same variables as in Step 2, write an equation to record your results for the shorter side.

Step 5 In this system the equations from Steps 2 and 4 have different coefficients for each variable. What can you do to one equation so that the variable C is eliminated when you add both equations?

Step 6 Use your answer to Step 5 to set up the addition of two equations. Once you eliminate the variable C, use the balancing method to solve for P.

Step 7 Substitute the value for P into one of the original equations to find C.

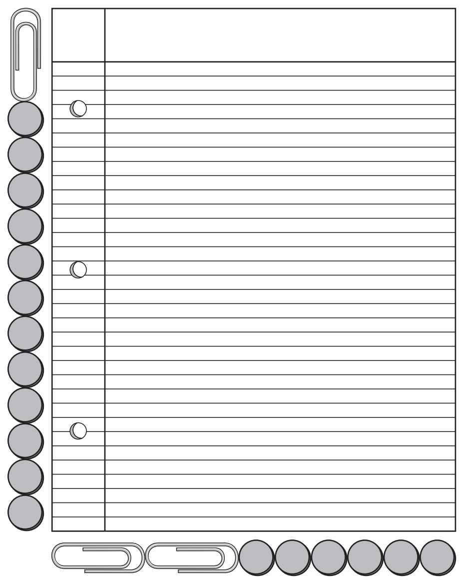
Step 8 Check that your solution satisfies both equations.

Step 9 Describe at least one other way to solve this system by elimination.

Step 10 Explain the real-world meaning of the solution. Describe other experiments in measuring that you can solve using a system of equations.

Name ________________ **Period** ________ **Date** ________

In this investigation you'll create a system of equations by using paper clips and pennies as variables.



Step 1 The diagram shows a piece of lined paper with a combination of large paper clips and pennies lining a short edge and a long edge.

Step 2 Use C for the length of one paper clip and P for the diameter of one penny. Write an equation in standard form for the length of the long side of the paper.

Step 3 Now you'll prepare to write the other equation for the system, again referring to the illustration.

Step 4 Using the same variables as in Step 2, write an equation in standard form for the length of the shorter side of the paper.

Step 5 In this system the equations from Steps 2 and 4 have different coefficients for each variable. What can you do to one equation so that the variable C is eliminated when you add both equations?

Step 6 Use your answer to Step 5 to set up the addition of two equations. Once you eliminate the variable *C*, use the balancing method to solve for *P*.

Step 7 Substitute the value for *P* into one of the original equations to find *C*.

Step 8 Check that your solution satisfies both equations.

Step 9 Describe at least one other way to solve this system by elimination.

Step 10 Explain the real-world meaning of the solution. Describe other experiments in measuring that you can solve using a system of equations.

Investigation • Diagonalization

Name ______________________________ **Period** __________ **Date** ______________

In this investigation you will see how to combine row operations in your solution process.

Consider the system of equations

$$\begin{cases} 2x + y = 11 \\ 6x - 5y = 9 \end{cases}$$

Step 1 Write the matrix for this system. What does the first row contain? The second row?

Step 2 Describe how to use row operations to get 0 as the first entry in the second row. Write this matrix.

Step 3 Next, get 1 as the second number in the second row of your matrix from Step 2.

Step 4 Use row operations on the matrix from Step 3 to get 0 as the second number in row 1.

Step 5 Next, get 1 as the first number of row 1 of your matrix from Step 4. Tell what this matrix means, and give the solution to the system.

Step 6 Check your solution using elimination. Eliminate *x* first.

Step 7 Look at the first three rules for Row Operations in a Matrix. How do they correspond to steps in the elimination process?

Investigation • Toe the Line

Name ______________________ **Period** __________ **Date** ______________

You will need: chalk or a tape measure to mark a segment

In this investigation you will analyze properties of inequalities and discover some interesting results.

First you'll act out operations on a number line.

Step 1 In your group, choose an announcer, a recorder, and two walkers. The two walkers make a number line on the floor with marks from −10 to 10. The "Operation" column in the table gives instructions to the walkers about how to move along the number line.

Operation	Walker A's position	Inequality symbol	Walker B's position
Starting number	2		4
Add 2			
Subtract 3			
Add −2			
Subtract −4			
Multiply by 2			
Subtract 7			
Multiply by −3			
Add 5			
Divide by −4			
Subtract 2			
Multiply by −1			

Procedure Note

The announcer calls out operations for Walkers A and B. The walkers perform operations on their numbers by walking to the resulting values on the number line. The recorder logs the position of each walker after each operation.

Step 2 Read the Procedure Note. As a trial, act out the first operation in the table: Walker A simply stands at 2 on the number line, and Walker B stands at 4.

Enter the inequality symbol into the table that describes the relative position of Walkers A and B on the number line. Be sure you have written a true inequality.

Step 3 Call out the operations. After the walkers calculate their new numbers, record the operation and walkers' positions in the next row.

Step 4 As a group, discuss which inequality symbol to enter into each cell of the third column.

Next you'll analyze what each operation does to the inequality.

Step 5 What happens to the walkers' relative positions on the number line when the operation adds or subtracts a positive number? A negative number? Does anything happen to the direction of the inequality symbol?

Step 6 What happens to the walkers' relative positions on the number line when the operation multiplies or divides by a positive number? Does anything happen to the inequality symbol?

Step 7 What happens to the walkers' relative positions on the number line when the operation multiplies or divides by a negative number? Does the inequality symbol change directions?

Investigation • Toe the Line (continued)

Step 8 Which operations on an inequality reverse the inequality symbol? Does it make any difference which numbers you use? Consider fractions and decimals as well as integers.

Step 9 Check your findings about the effects of adding, subtracting, multiplying, and dividing by the same number on both sides of an inequality by creating your own table of operations and walkers' positions.

Operation	Walker A's position	Inequality symbol	Walker B's position
Starting number			

Investigation • Graphing Inequalities

Name ______________________________ **Period** __________ **Date** ______________

You will need: the worksheet Graphing Inequalities Grids

i. $y \square 1 + 0.5x$ **ii.** $y \square -1 - 2x$

iii. $y \square 1 - 0.5x$ **iv.** $y \square 1 - 2x$

Step 1 Each member of the group should choose a different statement from above.

Step 2 Use the Graphing Inequalities Grids graph. Evaluate the right side of your statement for $x = -3$. For each circle in the first column on the graph, fill in $>$ if the y-value of the point is greater than your value, $=$ if the values are equal, and $<$ if the y-value is less than your value.

Step 3 Repeat Step 2 for $x = -2, -1, 0, 1, 2,$ and 3.

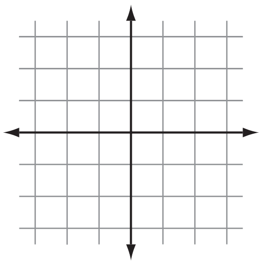
Next you'll analyze the results of your graph.

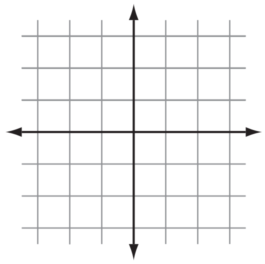
Step 4 What do you notice about the circles filled with the equal sign? Describe any other patterns you see.

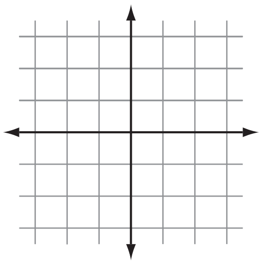
Step 5 Test a point with fractional or decimal coordinates that is not represented by a circle on the grid. Compare your result with the symbols on the same side of the line of equal signs as your point.

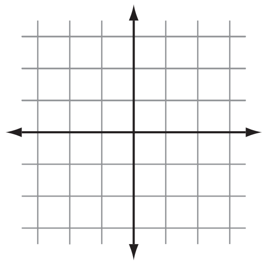
Step 6 The graphs on the next page are scaled from -3 to 3 on each axis. Under the graph, write your statement with the "less than" symbol, $<$. Shade the region of points that makes your statement true. If the points on the line make an inequality true, draw a solid line through them. If not, draw a dashed line. Repeat this step for each of the remaining symbols ($>$, $\leq$, $\geq$, $=$).

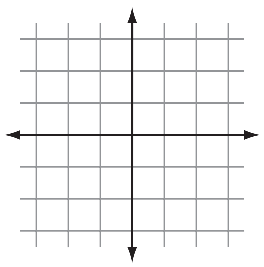
Investigation • Graphing Inequalities (continued)











Finally, you'll draw general conclusions by comparing graphs in your group.

Step 7 Compare your graphs with those of others in your group. What graphs require a solid line? A dashed line?

Step 8 What graphs require shading? Shading above the line? Below the line?

Step 9 Discuss how to use one point to check the graph of an inequality.

Investigation • A "Typical" Envelope

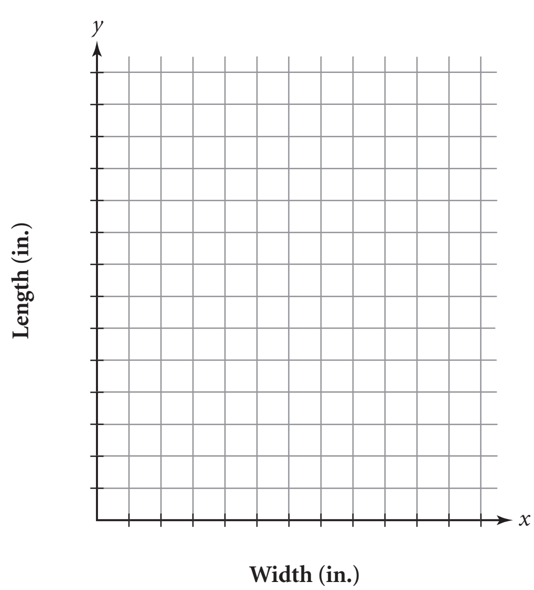
Name ______________________________ **Period** __________ **Date** ______________

The U.S. Postal Service imposes several constraints on the acceptable sizes for an envelope. One constraint is that the ratio of length to width must be less than or equal to 2.5, and another is that this ratio must be greater than or equal to 1.3.

Step 1 Define variables and write an inequality for each constraint.

Step 2 Solve each inequality for the variable representing length. Decide whether or not you have to reverse directions on the inequality symbols. Then write a system of inequalities to describe the Postal Service's constraints on envelope sizes.

Step 3 Decide on appropriate scales for each axis and label the axes. Decide if you should draw the boundaries of the system with solid or dashed lines. Graph each inequality on the axes. Shade each half-plane with a different color or pattern.

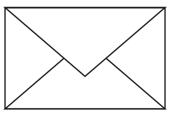


Investigation • A "Typical" Envelope (continued)

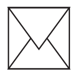
Step 4 Where on the graph are the solutions to the system of inequalities? Discuss how to check that your answer is correct.

Step 5 Decide if each envelope satisfies the constraints by locating the corresponding point on your graph.

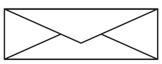
a. 5 in. by 8 in.



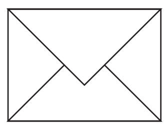
b. 3 in. by 3 in.



c. 2.5 in. by 7.5 in.



d. 5.5 in. by 7.5 in.



Step 6 Do the coordinates of the origin satisfy this system of inequalities? Explain the real-world meaning of this point. What constraints can you add to more realistically model the Postal Service's acceptable envelope sizes? How do these additions affect the graph?